Induction Motors Torque Control with Torque per Ampere Ratio Maximization

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Abstract

The paper reports new theoretical and experimental results in vector control of induction motors. A novel direct field-oriented torque tracking controller is designed using output-feedback linearizing procedure which guarantees asymptotic torque tracking and maximal torque per Ampere ratio during steady state. The efficiency improvement is obtained by adjusting the flux level according to optimization procedure of maximal torque per Ampere (MTA) ratio that is very close to the optimization criterion of minimum losses. Main advantage of MTA control is simplicity of practical implementation.
The proposed controller assures quite fast dynamics in the torque response and exponential stability. An intensive experimental investigations proof the effectiveness of the proposed control technique.

Key words: induction motor, field-oriented control, maximum torque per Ampere ratio, efficiency optimization.

1. INTRODUCTION

Modern electrical drives based on induction motors (IM) are the most spread electromechanical systems. From the control point of view, they represent a complex multivariable non-linear problem and they constitute an important area of application for non-linear control theory. On the other hand to solve the IM control problem achieving at the same time fast dynamics, high energy efficiency, robustness with respect of IM parameters variation and simple implementation is an important practical task.
Over the recent years several non-linear control strategies has been proposed to solve torque (speed) and flux tracking (regulation) problem. Among them based on output feedback [1], feedback linearization [2] and passivity based control [3].
Standard and advanced field-oriented vector controls of IM typically operate holding constant amplitude of the flux vector even at low values of produced torque. Such approach provides an ability to obtain the best dynamic performance, while the efficiency and power factor can be quite low especially at values of torque that is lower than rated one.
The efficiency improvement can be obtained by adjusting the flux level according to some optimization procedure. The regulation of the flux lowers the drive dynamic performance but this approach can be used in those applications where extremely fast response is not necessary, for example in electrical vehicles.
There are different control strategies that allow to minimize total losses, maximize power factor and others [4], [5]. The loss models are quite complex and therefore optimization usually performed numerically with the calculated optimum flux reference value stored in a look-up table or using on-line search algorithms. Both approaches have well known disadvantages in real time implementation such as parameter dependency of look-up table data and slow dynamics of search method. An alternative way to achieve the active losses minimization is known as a maximum torque per Ampere control. Few theoretical results based on modified field orientation are available [6], [7]. The main disadvantages of the known solutions are: pour torque dynamics [7], complexity of controller [6] with non-holonomy control strategy, inability to define required dynamic performance characteristics.
The main aim of this paper is to solve asymptotic torque tracking control problem while maximizing torque per Ampere during steady state conditions of operation. The approach proposed is based on dynamic output-feedback linearizing control using direct (based on the observer) field-orientation. Controller adjust the flux reference value increasing the efficiency at light loads, since maximization of torque per Ampere (MTA) ratio is very close to the optimization criterion of minimum losses. Main advantage of MTA control is simplicity of practical implementation.

2. PROBLEM STATEMENT

The standard two-phase model of symmetrical IM, under the assumptions of linear magnetic circuits and balanced operating conditions, is represented in an arbitrary rotating reference frame (d-q) as [1]

$$
\begin{aligned}
\dot{\omega} &= \frac{1}{J}\left(T - T_L\right),\\
T &= \mu_1\left(\psi_d i_q - \psi_q i_d\right),\\
\dot{i}_d &= -\gamma i_d + \omega_0 i_q + \alpha\beta\psi_d + \beta\omega\psi_q + \frac{u_d}{\sigma},\\
\dot{i}_q &= -\gamma i_q - \omega_0 i_d + \alpha\beta\psi_q - \beta\omega\psi_d + \frac{u_q}{\sigma},\\
\dot{\psi}_d &= -\alpha\psi_d + \left(\omega_0 - \omega\right)\psi_q + \alpha L_m i_d,\\
\dot{\psi}_q &= -\alpha\psi_q - \left(\omega_0 - \omega\right)\psi_d + \alpha L_m i_q,\\
\dot{\varepsilon}_0 &= \omega_0, \varepsilon_0\left(0\right) = 0,
\end{aligned}
\tag{1}
$$

where $\mathbf{u} = (u_d, u_q)^T$ is the control vector of stator voltages, $\mathbf{i} = (i_d, i_q)^T$, $\boldsymbol{\psi} = (\psi_d, \psi_q)^T$ denote stator current and rotor flux vectors, ω is the rotor speed, T is the electro-magnetic torque of the IM, and T_L is the load torque. Subscripts d and q stand for vector components in the (d-q) reference frame, ε_0 is the angular position of the (d-q) reference frame with respect to a fixed stator reference frame (a-b), where physical variables are defined.
Positive constants related to the electrical and mechanical parameters of the IM are defined as follows:

$$\alpha = \frac{R_r}{L_r},\ \mu_1 = \frac{3}{2}\frac{L_m}{L_r},\ \sigma = L_s\left(1 - \frac{L_m^2}{L_s L_r}\right),\ \beta = \frac{L_m}{\sigma L_r},\ \gamma = \frac{R_s}{\sigma} + \alpha L_m \beta\ ,$$

where R_s, R_r, L_s, L_r are stator/rotor resistances and inductances, respectively, L_m is the magnetizing inductance, J is the rotor inertia.
Let define the torque and flux reference trajectories as T^* and $\psi^* > 0$ respectively. The torque and flux tracking errors are

$$
\begin{aligned}
\tilde{T} &= T - T^*,\\
\tilde{\psi} &= \left|\psi\right| - \psi^*.
\end{aligned}
$$

Consider the IM model (1) and assume that:
A1. The stator currents and rotor speed are available for measurement.
A2. The motor parameters are known and constant.
A3. The torque and flux reference trajectories T^* and $\psi^* > 0$ are smooth and bounded functions together with the first time derivative.
Under these assumptions, it is required to design a controller satisfying the following control objectives:
CO1. Global asymptotic torque-flux tracking

$$\lim_{t\to\infty} \tilde{T} = 0$$

$$\lim_{t\to\infty} \tilde{\psi} = 0,$$

with all internal variables bounded.
CO2. Asymptotic decoupling of output variables via asymptotic field orientation, namely, if $\tilde{\psi}(0) = 0, \psi_q(0) = 0,$ then $\tilde{\psi}(t) \equiv 0, \forall t > 0$ and the dynamics of torque error are independent of the flux control.
CO3. Torque per Ampere maximization in static mode of operation which is achieved when [7]

$$i_d = \left|i_q\right| + \delta, \tag{2}$$

where δ is small positive constant which is used to avoid singularity when $T^* = 0$.

3. CONTROLLER DESIGN FOR CURRENT-FED IM

In current-fed IM $i_d = i_d^*, i_q = i_q^*$, where i_d^*, i_q^* are the reference trajectories for stator currents. First we define a reduced order flux observer for (1)

$$\begin{aligned} \dot{\hat{\psi}} &= -\alpha\hat{\psi} + \alpha L_m i_d, \\ \dot{\varepsilon}_0 &= \omega_0 = \omega + \frac{\alpha L_m i_q}{\hat{\psi}}, \end{aligned} \tag{3}$$

which guaranties that flux errors

$$\begin{aligned} \tilde{\psi}_d &= \psi_d - \hat{\psi}, \\ \tilde{\psi}_q &= \psi_q, \end{aligned} \tag{4}$$

dynamics, given by

$$\begin{aligned} \dot{\tilde{\psi}}_d &= -\alpha\tilde{\psi}_d + \omega_2\tilde{\psi}_q, \\ \dot{\tilde{\psi}}_q &= -\alpha\tilde{\psi}_q - \omega_2\tilde{\psi}_d, \\ \omega_2 &= \omega_0 - \omega, \end{aligned} \tag{5}$$

is exponentially stable, when $\hat{\psi} > 0$.

For current-fed IM, when current i_d is assumed to be the control input in the flux equations, we define:

$$i_d = L_m^{-1}\left(\psi_0^* + L_m\left|i_q\right|\right) > 0, \tag{6}$$

where $\psi_0^* > 0$ – minimal level of flux which is necessary to avoid singularity in (3). From (6) it follows that $i_d > 0$, and therefore all solutions of the first equation in (3) are positive functions $\hat{\psi}(t) > 0, \hat{\psi}(0) = \psi_0^*$.

By using previously defined flux errors (4), the electromagnetic torque equation of IM in (1) becomes

$$T = \mu_1\left(\psi_d i_q - \tilde{\psi}_q i_d\right) \tag{7}$$

After some computations torque dynamics can be presented as

$$\begin{aligned} \dot{T} &= -\alpha T + \alpha\mu_1\left(\psi_0^* + L_m\left|i_q\right|\right)i_q + \mu_1\hat{\psi}\dot{i}_q + \tilde{\varphi}(t,\tilde{\psi}_d,\tilde{\psi}_q), \\ \tilde{\varphi}(t,\tilde{\psi}_d,\tilde{\psi}_q) &= \mu_1\left[\tilde{\psi}_d\left(\dot{i}_q + \omega_2 i_d\right) - \tilde{\psi}_q\left(\dot{i}_d - \omega_2 i_q\right)\right] \end{aligned}$$

or in the error form

$$\dot{\tilde{T}} = -\alpha T^* - \dot{T}^* + \alpha\mu_1\left(\psi_0^* + L_m\left|i_q\right|\right)i_q + \mu_1\hat{\psi}\dot{i}_q - \alpha\tilde{T} + \tilde{\varphi}\left(t,\tilde{\psi}_d,\tilde{\psi}_q\right) \tag{8}$$

Define the following dynamic output feedback linearizing controller for (8)

$$\dot{i}_q = -\frac{\alpha}{\hat{\psi}}\left(\psi_0^* + L_m\left|i_q\right|\right)i_q + \frac{1}{\mu_1\hat{\psi}}\left[\alpha T^* + \dot{T}^*\right] \tag{9}$$

Total torque-flux error dynamic with this torque controller is given by

$$\dot{\tilde{T}} = -\alpha\tilde{T} + \tilde{\varphi}_1\left(t,\tilde{\psi}_d,\tilde{\psi}_q\right) \tag{10}$$

$$\begin{aligned} \dot{\tilde{\psi}}_d &= -\alpha\tilde{\psi}_d + \omega_2\tilde{\psi}_q, \\ \dot{\tilde{\psi}}_q &= -\alpha\tilde{\psi}_q - \omega_2\tilde{\psi}_d, \end{aligned} \tag{11}$$

$$\tilde{\varphi}_1\left(t,\tilde{\psi}_d,\tilde{\psi}_q\right)=\left(\alpha T^*+\dot{T}^*\right)\left[\frac{\tilde{\psi}_d}{\hat{\psi}}-\frac{\tilde{\psi}_q}{\hat{\psi}}sign(i_q)\right]+\frac{\tilde{\psi}_q}{\hat{\psi}}\alpha\mu_1\left(\left|i_q\right|\psi_0^*+2L_m i_q^2\right),$$

$$\dot{\hat{\psi}}=-\alpha\left|\hat{\psi}\right|+\alpha\psi_0^*+\alpha L_m\left|i_q\right|,$$
$$\dot{i}_q=-\frac{\alpha}{\hat{\psi}}\left(\psi_0^*+L_m\left|i_q\right|\right)i_q+\frac{1}{\mu_1\hat{\psi}}\left[\alpha T^*+\dot{T}^*\right].$$

References $T^*,\dot{T}^*$ are bounded functions by definition and therefore, $\dot{i}_q, i_q$, are bounded as well, i_d and $\hat{\psi}$ are positive. Two linear subsystems (10) and (11) are connected in series. Since $\lim_{t\to\infty}\left(\tilde{\psi}_d,\tilde{\psi}_q\right)=0$, $T^*,\dot{T}^*,\dot{i}_q,i_q,i_d$ are bounded, we conclude that $\lim_{t\to\infty}\tilde{\varphi}\left(t,\tilde{\psi}_d,\tilde{\psi}_q\right)=0$ and $\tilde{T}$ in (10) exponentially decays to zero. From this analysis it follows that control objectives CO.1 and CO.2 are globally achieved.

4. FULL ORDER CONTROLLER DESIGN

In actual IM drives, currents i_d and i_q in (6), (9), are not available as control inputs and can only represent theirs desired dynamics i_d^* and i_q^*. Stator voltages u_d and u_q are the available control inputs. Using back-stepping design procedure, the reduced-order controller is extended to the full-order case by adding a current loop. The outputs of the flux and torque controllers, corresponding to the right-hand side of (6) and solution of (9) respectively, are considered as the reference inputs i_d^* and i_q^* for inner current loops.
By defining the current error dynamics

$$\begin{aligned}\tilde{i}_d&=i_d-i_d^*\\ \tilde{i}_q&=i_q-i_q^*\end{aligned},\tag{12}$$

and using equations (1) the current-error dynamics becomes

$$\begin{aligned}\dot{\tilde{i}}_d&=-\gamma\tilde{i}_d+\omega_0 i_q+\alpha\beta\tilde{\psi}_d+\beta\omega\tilde{\psi}_q+\frac{u_d}{\sigma}+\alpha\beta\hat{\psi}-\gamma i_d^*-\dot{i}_d^*,\\ \dot{\tilde{i}}_q&=-\gamma\tilde{i}_q-\omega_0 i_d+\alpha\beta\tilde{\psi}_q-\beta\omega\tilde{\psi}_d+\frac{u_q}{\sigma}+\beta\omega\hat{\psi}-\gamma i_q^*-\dot{i}_q^*,\end{aligned}\tag{13}$$

The following control algorithm is designed for the d and q-axes current loops:

$$u_d=\sigma\left(\gamma i_d^*-\omega_0 i_q-\alpha\beta\hat{\psi}+\dot{i}_d^*-k_{id1}\tilde{i}_d\right)\tag{14}$$

$$\begin{aligned}u_q&=\sigma\left(\gamma i_q^*+\omega_0 i_d+\beta\omega\hat{\psi}+\dot{i}_q^*-k_{iq1}\tilde{i}_q+x_q\right),\\ \dot{x}_q&=-k_{iiq}\tilde{i}_q,\end{aligned}\tag{15}$$

where $k_{id1}>0$, $k_{iq1}>0$ are the current controllers proportional gains respectively, $k_{iiq}>0$ is the torque current component controller integral gain.
Let us consider the modified reduced order flux observer given by

$$\begin{aligned}\dot{\hat{\psi}}&=-\alpha\hat{\psi}+\alpha L_m i_d,\\ \dot{\varepsilon}_0&=\omega_0=\omega+\frac{\alpha L_m i_q}{\hat{\psi}}+\frac{v_\varepsilon}{\hat{\psi}},\end{aligned}\tag{16}$$

where v_ψ, v_ε are the correction terms to be defined later.
From (1), (14) and (16) the flux-current error dynamics can be written as

$$\begin{aligned}\dot{\tilde{\psi}}_d &= -\alpha\tilde{\psi}_d + \omega_2\tilde{\psi}_q,\\ \dot{\tilde{\psi}}_q &= -\alpha\tilde{\psi}_q - \omega_2\tilde{\psi}_d - v_\varepsilon,\end{aligned} \tag{17}$$

$$\dot{\tilde{i}}_d = -k_{id}\tilde{i}_d + \alpha\beta\tilde{\psi}_d + \beta\omega\tilde{\psi}_q, \tag{18}$$

$$\begin{aligned}\dot{\tilde{i}}_q &= -k_{iq}\tilde{i}_q + \alpha\beta\tilde{\psi}_q - \beta\omega\tilde{\psi}_d + x_q,\\ \dot{x}_q &= -k_{iiq}\tilde{i}_q,\end{aligned} \tag{19}$$

where $k_{id} = \gamma + k_{id1}$, $k_{iq} = \gamma + k_{iq1}$.
To design the correcting term in (16) we consider the following quadratic function

$$V = \frac{1}{2}\left[\tilde{\psi}_d^2 + \tilde{\psi}_q^2 + \lambda\tilde{i}_d^2\right] > 0\,,\;\; \lambda > 0 \tag{20}$$

The time derivative of V along the trajectories of (17)-(19) is

$$\dot{V} = -\alpha\tilde{\psi}_d^2 - \alpha\tilde{\psi}_q^2 - \lambda k_{id}\tilde{i}_d^2 + \lambda\alpha\beta\tilde{i}_d\tilde{\psi}_d - v_\varepsilon\tilde{\psi}_q + \lambda\beta\omega\tilde{i}_d\tilde{\psi}_q \tag{21}$$

Selecting the correction term v_ε as

$$v_\varepsilon = \lambda\beta\omega\tilde{i}_d \tag{22}$$

the following expression results for the $\dot{V}$

$$\dot{V} = -\alpha\tilde{\psi}_d^2 - \alpha\tilde{\psi}_q^2 - \lambda k_{id}\tilde{i}_d^2 + \lambda\alpha\beta\tilde{i}_d\tilde{\psi}_d < 0,\; if\;\; k_{id} > \frac{\lambda\alpha\beta^2}{4} \tag{23}$$

From (20) and (23) it can be concluded that equilibrium point of system (17) – (18)

$$\left(\tilde{\psi}_d, \tilde{\psi}_q, \tilde{i}_d\right)^T = 0 \tag{24}$$

is globally exponential stable. Then from the analysis of (19) it follows that equilibrium point

$$\left(\tilde{i}_q, x_q\right)^T = 0 \tag{25}$$

is globally exponentially stable too.
The torque-error dynamics using the definition for i_d^* and i_q^* becomes

$$\begin{aligned}\dot{\tilde{T}} = &-\alpha\tilde{T} + \alpha\mu_1\left(\tilde{i}_d i_q^* + i_d^*\tilde{i}_q + \tilde{i}_d\tilde{i}_q\right) + \mu_1\hat{\psi}\dot{\tilde{i}}_q + \mu_1\tilde{\psi}_d\left[\left(\dot{\tilde{i}}_q + \dot{i}_q^*\right) + \omega_2\left(\tilde{i}_d + i_d^*\right)\right] -\\ &-\mu_1\tilde{\psi}_q\left[\left(\dot{\tilde{i}}_d + \dot{i}_d^*\right) - \omega_2\left(\tilde{i}_q + i_q^*\right)\right] + \mu_1\lambda\left[\left(\beta\omega\tilde{i}_d + \alpha\beta\tilde{i}_q\right)\left(\tilde{i}_d + i_d^*\right)\right],\end{aligned} \tag{26}$$

$$\omega_2 = \frac{\alpha L_m\left(\tilde{i}_q + i_q^*\right)}{\hat{\psi}} + \frac{\lambda\beta\omega\tilde{i}_d}{\hat{\psi}} \tag{27}$$

Considering that i_d^* and i_q^* are bounded, solutions of (17)-(19) are globally exponentially stable, then $\tilde{T}$ goes exponentially to zero. Thus, the asymptotic torque tracking together with steady state torque per Ampere maximization are achieved.
The block diagram of the proposed torque-flux controller is presented on Figure 1.

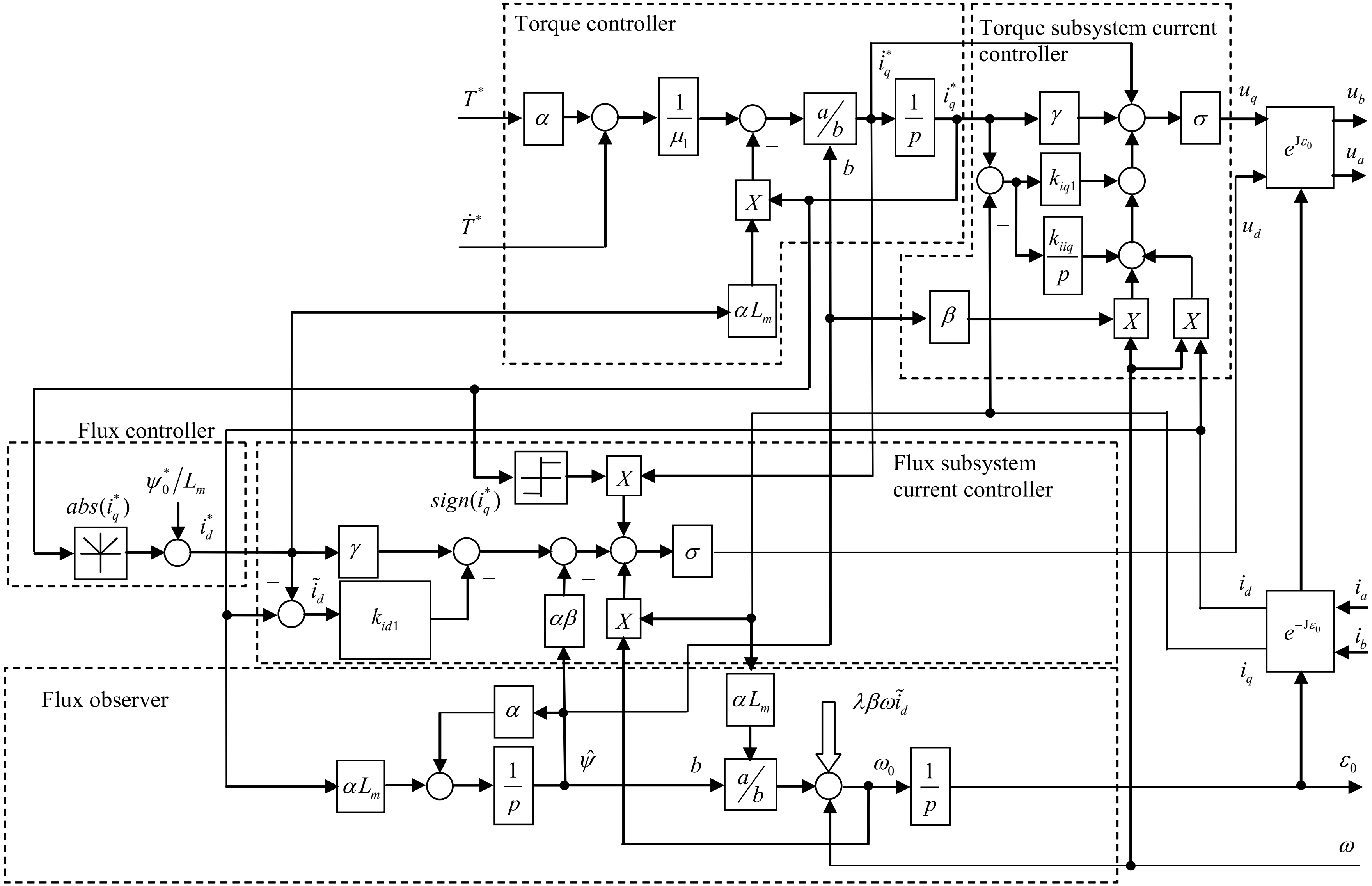

Figure 1 – The block diagram of the direct field-oriented controller with torque per Ampere maximization

5. EXPERIMENTAL RESULTS

The experimental investigation of designed controller (6), (9), (14) – (16) was performed using the rapid prototyping station [8]. The sampling time was set at $200\ \mu s$ and the PWM frequency of power converter was defined as $10\ kHz$.

Dynamics of the torque controller has been studied under constant speed, provided by the standard speed controlled induction motor (coupled with the motor under investigation). The indirect limitation of the rotor flux vector modulus was introduced by limiting the flux component of the stator current i_d on the rated level.

Experimental tests have been performed using 2.2kW induction motor whose parameters are reported in Appendix A. During the tests the following operating sequences were used:

1. The initial time interval $(0 \le t < 0.1\ s)$ is used to maintain the minimum flux reference $(\psi_0^* = 0.05\ Wb)$.
2. At time $t = 0.1\ s$ the torque reference reported in Figure 2, reaching the torque of $5\ Nm$ (33% of rated), is applied.
3. At time $t = 0.6\ s$ the torque reference starts to increase to the value of $10\ Nm$ (66% of rated).
4. At time $t = 1.1\ s$ the torque reference reaches the rated value of $15\ Nm$.
5. During the time interval $1.95\ s < t < 3.15\ s$ sinusoidal torque reference is applied.

Controller gains during the experiments were set to: $k_{id1} = 800$, $k_{iq1} = 800$, $k_{iiq} = 160000$, $\lambda = 0.02$.

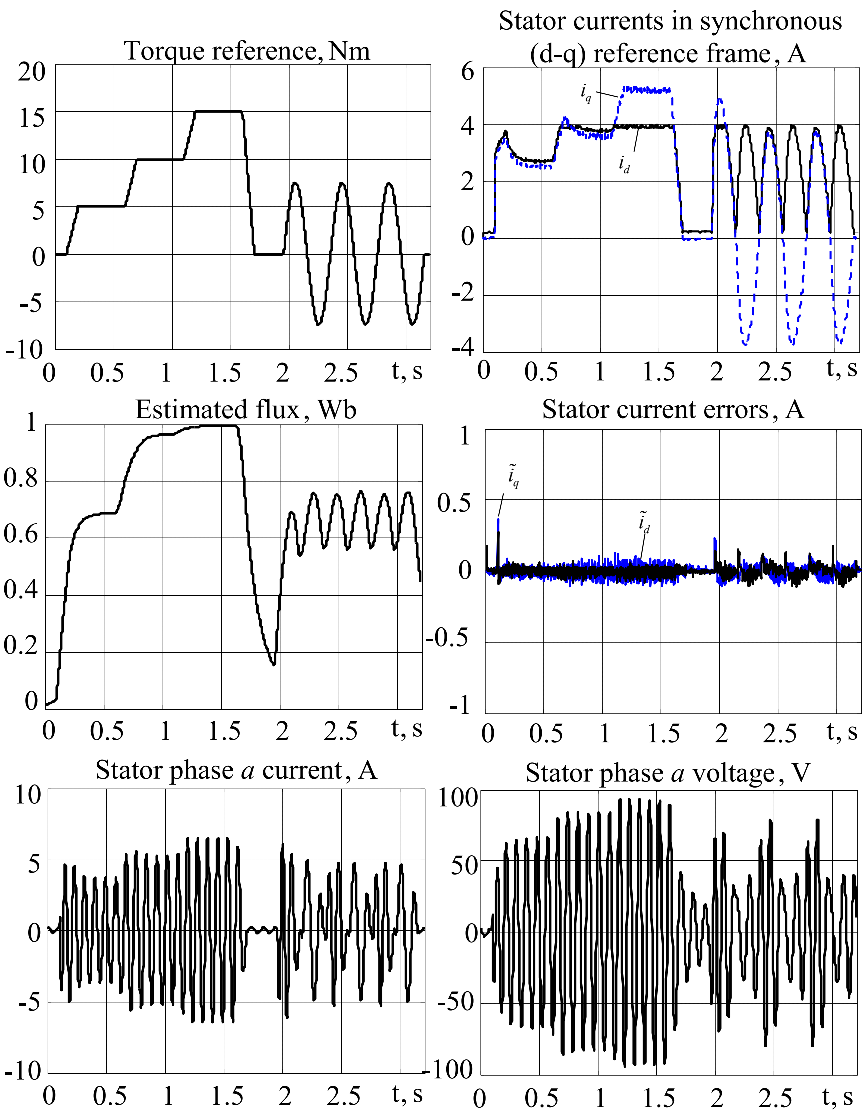

Figure 2 – Transients during the tracking of general torque reference.

Transients reported on Fig.2 demonstrate the torque tracking performance of the proposed controller. From the trajectories of stator currents represented in synchronous reference frame $(d-q)$ it can be seen that flux and torque components are nearly equal that assures the maximization of torque per Ampere ratio if there are no limitations for flux current. When the torque reference is more than $10\ Nm$ the limitation of flux current component is applied to prevent the saturation of IM magnetic circuit. Almost zero stator current errors show an excellent tracking performance of the current control. From dynamic behavior of current errors we conclude that flux estimation errors converge to zero and therefore asymptotic torque tracking is achieved for torque references (positive and negative), while torque/current optimization is possible only when flux reference current is less than rated one.
From the estimated flux trajectory it can be seen that value of rotor flux is varying according to variations of the torque reference value. Since MTA control strategy is very close to the optimization criterion of minimum losses we conclude that increasing of the energy efficiency can be achieved at light loads. The output current and voltage of phase a in stator reference frame $(a-b)$ are also shown.

CONCLUSION

The novel direct field-oriented control algorithm of induction motor is designed using output-feedback linearizing procedure which guarantees global asymptotic torque tracking and maximal torque per Ampere ratio during steady state. Controller adjust the flux reference value increasing the efficiency at light loads, since maximization of torque per Ampere (MTA) ratio is very close to the optimization criterion of minimum losses.
The proposed controller assures quite fast dynamics in the torque response and exponential stability. An intensive experimental investigations proof the effectiveness of the proposed control technique.

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APPENDIX A

Rated parameters of the induction motor used in experiments

Description	Parameter	Value	Units
Rated torque	M_{rated}	15	Nm
Rated stator current modulus	$\lvert i_s \rvert_{rated}$	7	A
Stator voltage modulus	$\lvert u_s \rvert_{rated}$	311	V
Rotor flux norm	ψ_{rated}	0.99	Wb
Rated rotor shaft speed	ω_{rated}	151.76	rad / s
Stator resistance	R_s	3.2	Ω
Rotor resistance	R_r	2.1	Ω
Stator (rotor) inductance	$L_s = L_r$	0.2655	H
Mutual inductance	L_m	0.257	H
Rotor inertia	J	0.0165	$Kg\ m^2$
Number of pole pairs	p_n	2	–